EXISTENCE AND UNIQUENESS OF ANALYTIC SOLUTIONS OF THE SHABAT EQUATION

EUGENIA N. PETROPOULOU

Received 30 May 2004

Sufficient conditions are given so that the initial value problem for the Shabat equation has a unique analytic solution, which, together with its first derivative, converges absolutely for $z \in \mathbb{C} : |z| < T$, $T > 0$. Moreover, a bound of this solution is given. The sufficient conditions involve only the initial condition, the parameters of the equation, and T. Furthermore, from these conditions, one can obtain an upper bound for T. Our results are in consistence with some recently found results.

1. Introduction and main results

Consider the nonlinear functional differential equation

$$f'(z) + q^2 f'(qz) + f^2(z) - q^2 f^2(qz) = \mu, \tag{1.1}$$

$$f(0) = f_0, \tag{1.2}$$

where q, μ, and f_0 are in general complex numbers. Equation (1.1) for $q = 1/k$, $0 < k < 1$, and $\mu = 1 - (1/k^2)$ was derived by Shabat [10] when he considered the similarity solution of the dressing dynamical system

$$(f_j + f_{j+1})_x = f_j^2 - f_{j+1}^2 + \lambda_j - \lambda_{j+1}, \quad j = 0, \pm 1, \pm 2, \ldots, \tag{1.3}$$

which is closely interconnected with the spectral theory of the linear Schrödinger equation

$$\psi_{xx} + [q(x) + \lambda]\psi = 0. \tag{1.4}$$

Equation (1.1) is studied for $|q| < 1$, because if $|q| > 1$, then (1.1) is equivalent with

$$\Phi'(w) + p^2\Phi'(pw) + \Phi^2(w) - p^2\Phi^2(pw) = -\mu p^2 \tag{1.5}$$

after setting

$$f(z) = -\Phi(pw), \qquad qz = w, \qquad p = \frac{1}{q}. \tag{1.6}$$

Abstract and Applied Analysis 2005:8 (2005) 855–862
DOI: 10.1155/AAA.2005.855