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Research Article

On the Complex Zeros of Some Families of Orthogonal Polynomials

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The complex zeros of the orthogonal Laguerre polynomials $L_n^{(a)}(x)$ for $a < -n$, ultraspherical polynomials $P_n^{(\lambda)}(x)$ for $\lambda < -n$, Jacobi polynomials $P_n^{(a,\beta)}(x)$ for $a < -n$, $\beta < -n$, $a + \beta < -2(n+1)$, orthonormal Al-Salam-Carlitz II polynomials $P_n^{(a)}(x;q)$ for $a < 0$, $0 < q < 1$, and q-Laguerre polynomials $L_n^{(a)}(x;q)$ for $a < -n$, $0 < q < 1$ are studied. Several inequalities regarding the real and imaginary properties of these zeros are given, which help locating their position. Moreover, a few limit relations regarding the asymptotic behavior of these zeros are proved. The method used is a functional analytic one. The obtained results complement and improve previously known results.

1. Introduction

Orthogonal polynomials appear naturally in various problems of physics and mathematics and are considered as one of the basic tools in confronting problems of mathematical physics. Also, orthogonal polynomials have many important applications in problems of numerical analysis, such as interpolation or optimization. For a survey on applications and computational aspects of orthogonal polynomials, see [1] and the references therein.

Some of the most important properties of orthogonal polynomials, $P_n(x)$, are the following.

(P1) The orthogonal polynomials $P_n(x)$ are orthogonal with respect to a weight function $\varrho(x) > 0$ on an interval of orthogonality $[a,b] \subseteq \mathbb{R}$ and all their zeros are real and simple and lie inside (a,b).

(P2) Some classes of orthogonal polynomials $P_n(x)$ (including some of the classes studied in the present paper) satisfy an ordinary differential equation of the form

$$\sigma(x)P_n''(x) + \tau(x)P_n'(x) + \lambda_n P_n(x) = 0, \tag{1.1}$$