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Bounded Solutions and Asymptotic Stability of Nonlinear Difference Equations in the Complex Plane II

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Abstract—A functional analytic method is used to prove a theorem which establishes the existence and the uniqueness of the solution of a nonlinear difference equation in the Banach space l_1. The boundedness of the solution and the local asymptotic stability of the equilibrium points of the nonlinear difference equation under discussion are derived immediately from this theorem. The proof of the theorem has a constructive character which allows us to obtain information about the size of the region of attraction of the equilibrium points. Some known nonlinear difference equations are studied as particular cases of the theorem. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords—Difference equations, Bounded solutions, Asymptotic stability.

1. INTRODUCTION

In this paper, we study the m^{th} order nonhomogeneous, nonlinear difference equation of the form

$$\begin{aligned} f(n+m) + \sum_{p=1}^{m} (\alpha_p + \beta_p(n)) f(n+m-p) = g(n) + \sum_{s=2}^{\infty} c_s(n) [f(n+q)]^s \\ + \sum_{i=1}^{N} \sum_{k=1}^{\infty} d_{ik}(n) \left[f(n+q_{i1}) f(n+q_{i2})\right]^k \\ + \sum_{t=1}^{\Lambda} \sum_{k=1}^{\infty} b_{tk}(n) \left[f(n+q_{t3}) f(n+q_{t4}) f(n+q_{t5})\right]^k \\ + \sum_{j=1}^{M} \sum_{k=1}^{\infty} l_{jk}(n) \left[A_j f(n+q_{j6}) + B_j f(n+q_{j7})\right]^k f(n+q_{j8}), \end{aligned} \tag{1.1}$$

where m, N, M, Λ are positive integers, q, q_{i1}, q_{i2}, $i = 1, \dots, N$, q_{t3}, q_{t4}, q_{t5}, $t = 1, \dots, \Lambda$, q_{j6}, q_{j7}, q_{j8}, $j = 1, \dots, M$ are nonnegative integers, α_p, $p = 1, \dots, m$ in general complex numbers, with the initial conditions

$$f(p) = u_p, \qquad p = 1, \dots, m. \tag{1.2}$$

Under suitable assumptions, on the complex sequences $\beta_p(n)$, $p = 1, \dots, m$, $c_s(n)$, $s = 2, 3, \dots$, $d_{ik}(n)$, $i = 1, \dots, N$, $l_{jk}(n)$, $j = 1, \dots, M$, $b_{tk}(n)$, $t = 1, \dots, \Lambda$, $k = 1, 2, \dots$, on the roots of the

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