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Solutions of Nonlinear Delay and Advanced Partial Difference Equations in the Space $l^1_{\mathbb{N}\times\mathbb{N}}$

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Abstract—A functional analytic method is used to prove a general theorem which establishes the existence and the uniqueness of a solution of a class of nonlinear delay and advanced partial difference equations in the Banach space $l^1_{\mathbb{N}\times\mathbb{N}}$. The proof of the theorem has a constructive character, which enables us to obtain a bound of the solution and a region, depending on the initial conditions and the parameters of the equation under consideration, where this solution holds. Some known nonlinear partial difference equations, which appear in applications, are studied as particular cases of the theorem. © 2003 Elsevier Science Ltd. All rights reserved.

Keywords—Nonlinear partial difference equations, Bounded solutions.

1. INTRODUCTION

In this paper, we study a class of nonlinear, nonhomogeneous, partial difference equations of the form

$$\begin{aligned} &\sum_{n=1}^{N} \alpha_n(i,j)u(i-\sigma_{n1}, j-\tau_{n1}) + \sum_{m=1}^{M} b_m(i,j)u(i+\sigma_{m2}, j+\tau_{m2}) \\ &\quad + \sum_{k=1}^{K} c_k(i,j)u(i-\sigma_{k3}, j+\tau_{k3}) + \sum_{l=1}^{\Lambda} d_l(i,j)u(i+\sigma_{l4}, j-\tau_{l4}) \\ &= p(i,j) + \sum_{s=2}^{\infty} f_s(i,j)[u(i+\sigma, j+\tau)]^s + \sum_{t=1}^{T} q_t(i,j)u(i+\sigma_{t5}, j+\tau_{t5})u(i+\sigma_{t6}, j+\tau_{t6}), \end{aligned} \tag{1.1}$$

where $i = \max_{n,k}\{\sigma_{n1}, \sigma_{k3}\}+1, \dots, j = \max_{n,l}\{\tau_{n1}, \tau_{l4}\}+1, \dots$, N, M, K, Λ, T are positive finite integers, σ, τ, σ_{n1}, τ_{n1}, σ_{m2}, τ_{m2}, σ_{k3}, τ_{k3}, σ_{l4}, τ_{l4}, σ_{t5}, τ_{t5}, σ_{t6}, τ_{t6}, $1 \le n \le N$, $1 \le m \le M$, $1 \le k \le K$, $1 \le l \le \Lambda$, $1 \le t \le T$ are nonnegative finite integers.

Under suitable assumptions on the complex sequences $\alpha_n(i,j)$, $b_m(i,j)$, $c_k(i,j)$, $d_l(i,j)$, $f_s(i,j)$, $g_t(i,j)$, $1 \le n \le N$, $1 \le m \le M$, $1 \le k \le K$, $1 \le l \le \Lambda$, $s = 2, 3, \dots$, $1 \le t \le T$, and also under

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