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POLYNOMIAL SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper it is proved that the condition

$\lambda = a_1(n-2)(n-1)+\gamma_1(m-2)(m-1)+\beta_1(n-1)(m-1)+\delta_1(n-1)+\epsilon_1(m-1)$,

where $n = 1, 2, ..., N$, $m = 1, 2, ..., M$ is a necessary and sufficient condition for the linear partial differential equation

$$(a_1x^2 + a_2x + a_3)u_{xx} + (\beta_1xy + \beta_2x + \beta_3y + \beta_4)u_{xy} \\ +(\gamma_1y^2 + \gamma_2y + \gamma_3)u_{yy} + (\delta_1x + \delta_2)u_x + (\epsilon_1y + \epsilon_2)u_y = \lambda u,$$

where a_i, β_j, γ_i, δ_s, ϵ_s, $i = 1, 2, 3$, $j = 1, 2, 3, 4$, $s = 1, 2$ are real or complex constants, to have polynomial solutions of the form

$$u(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{nm}x^{n-1}y^{m-1}.$$

The proof of this result is obtained using a functional analytic method which reduces the problem of polynomial solutions of such partial differential equations to an eigenvalue problem of a specific linear operator in an abstract Hilbert space. The main result of this paper generalizes previously obtained results by other researchers.

1. Introduction.

The aim of this paper is the establishment of necessary and sufficient conditions so that the linear partial differential equation

$$(a_1x^2 + a_2x + a_3)u_{xx} + (\beta_1xy + \beta_2x + \beta_3y + \beta_4)u_{xy} + (\gamma_1y^2 + \gamma_2y + \gamma_3)u_{yy} \\ +(\delta_1x + \delta_2)u_x + (\epsilon_1y + \epsilon_2)u_y = \lambda u, \quad (1)$$

where a_i, β_j, γ_i, δ_s, ϵ_s, $i = 1, 2, 3$, $j = 1, 2, 3, 4$, $s = 1, 2$ are real or complex constants, to have polynomial solutions of the form

$$u(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{nm}x^{n-1}y^{m-1}. \quad (2)$$

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