Available online at www.sciencedirect.com
ScienceDirect
Chaos, Solitons and Fractals 33 (2007) 94–108
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Analytic bounded travelling wave solutions of some nonlinear equations ☆

Eugenia N. Petropoulou [a,*], Panayiotis D. Siafarikas [b]

[a] Department of Engineering Sciences, Division of Applied Mathematics and Mechanics, University of Patras, 26500 Patras, Greece
[b] Department of Mathematics, University of Patras, 26500 Patras, Greece

Accepted 23 January 2006

Abstract

By use of a functional analytic method it is proved that a general class of second order nonlinear differential equations has analytic bounded solution of the form $g(\xi)=\sum_{n=1}^{\infty}A_n(\frac{\xi}{T})^{n-1},\ |\xi|<T,\ T>0$. Such a solution is determined in a unique way, once the initial values $g(0)$ and $g'(0)$ are given, by a recurrence relation that the coefficients A_n satisfy. This general class includes the Lienard equation as well as an equation related to the Burgers–KdV equation, both of which are derived when seeking travelling wave solutions of the corresponding partial differential equations. By the method used in this paper all the solutions of these two equations that were found in two recent papers, are also derived here. Moreover, it is proved that they are analytic, absolutely convergent and a bound for each one of them is provided.

1. Introduction

During the last decades much attention has been given and many papers have appeared regarding travelling wave solutions and solitary solutions (which are travelling waves with some additional characteristics) of partial differential equations. This is due not only to strictly mathematical reasons, but also to many and significant physical problems where such kind of solutions are observed or desired.

When one seeks for travelling wave solutions u of a suitable partial differential equation of e.g. two variables: the spatial variable x and the time variable t, they seek for solutions of the form

$$u(x,t)=g(x-vt)=g(\xi), \tag{1.1}$$

where $\xi=x-vt$ and the constant v denotes the speed of the wave propagation. In this way the partial differential equation is reduced to an ordinary differential equation of the unknown function $g(\xi)$.

For example consider the Burgers–KdV equation

$$u_t+cuu_x+Gu_{xx}+su_{xxx}=0,\quad u=u(x,t), \tag{1.2}$$

☆ This work was supported by the EPEAEK Program "Pythagoras II".
* Corresponding author.
E-mail addresses: jenpetro@des.upatras.gr (E.N. Petropoulou), panos@math.upatras.gr (P.D. Siafarikas).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2006.01.078