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ON SOME DIFFERENCE EQUATIONS WITH EXPONENTIAL NONLINEARITY

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In memory of Professor Evangelos K. Ifantis

Abstract. The problem of the existence of complex ℓ_1 solutions of two difference equations with exponential nonlinearity is studied, one of which is nonautonomous. As a consequence, several information are obtained regarding the asymptotic stability of their equilibrium points, as well as the corresponding generating function and $z-$transform of their solutions. The results, which are obtained using a general theorem based on a functional-analytic technique, provide also a rough estimate of the region of attraction of each equilibrium point for the autonomous case. When restricted to real solutions, the results are compared with other recently published results.

1. **Introduction.** It is well known that difference equations arise naturally in the study of population models. Many times such kind of equations involve exponential terms. Indicatively the papers [9], [10], [13] and [14] are mentioned, where difference equations and systems of difference equations with exponential nonlinear terms are studied. However, even first order equations can be of great interest not only for population dynamics, but also from a mathematical point of view. Many of them exhibit complex dynamic behavior, even chaotic, despite their seemingly simple form, such as the logistic equation or the Ricker equation (see, e.g. [8]). Especially the Ricker equation, which was introduced in [12], has not only been the topic of many studies, but also has been generalized in various ways. For example in [1], an $n-$species Ricker model was studied with respect to the coexistence or extinction of one species. Also, the Ricker equation was combined with other type of equations, such as the Beverton-Holt equation, in [5] and [7] and the dynamics of the resulting system of difference equations were thoroughly studied.

More recently, in [2], the nonautonomous difference equation

$$u(k+1) = u(k)e^{(a-bu(k))(\theta_{k+1}-\theta_k)}, \tag{1}$$

which is of the Ricker type, was connected with the generalized logistic type differential equation

$$x'(t) = (a - bx(\beta(t)))x(t), \tag{2}$$

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