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ANALYTIC SOLUTIONS OF SOME NON-LINEAR ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT. A functional analytic method is used, so as to prove that certain non-linear ordinary differential equations have a unique solution, which together with some of its derivatives converges absolutely in a specified disc of the complex plane. Moreover, bounds of the solution and some of its derivatives are given, together with a region, depending on the initial conditions and the parameters of the equation, where such a solution holds. The method is applied to a general non-linear differential equation, which includes the Falkner-Skan and Blasius equations, to the Chazy equation, to the reduced Fisher equation and to the FitzHugh-Nagumo system.

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1. INTRODUCTION

In this paper, we study some specific non-linear ordinary differential equations, which appear in various physical problems. More precisely, we seek for solutions of them of the form:

$$f(z) = \sum_{n=1}^{\infty} \alpha_n z^{n-1}, \tag{1.1}$$

which (together with some of its derivatives) converge absolutely in a region $|z| < r$, $z \in \mathbb{C}$, $r > 0$.

The problem of finding power-series solutions or analytic solutions of differential equations is one of the oldest and most important problems in the theory of differential equations. One classical method for finding a solution (of the form (1.1)) of a linear differential equation is the Frobenius method. With this method, one assumes that the solution of the differential equation under consideration has the form (1.1) and then he substitutes (1.1) in the differential equation, in order to determine the coefficients

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