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ANALYTIC SOLUTIONS OF A CLASS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. We study a class of nonlinear partial differential equations, which can be connected with wave-type equations and Laplace-type equations, by using a functional-analytic technique. We establish primarily the existence and uniqueness of bounded solutions in the two-dimensional Hardy-Lebesque space of analytic functions with independent variables lying in the open unit disc. However these results can be modified to expand the domain of definition. The proofs have a constructive character enabling the determination of concrete and easily verifiable conditions, and the determination of the coefficients appearing in the power series solution. Illustrative examples are given related to the sine-Gordon equation, the Klein-Gordon equation, and to equations with nonlinear terms of algebraic, exponential and logistic type.

1. Introduction

Recently in [17], a functional-analytic technique was employed for the study of bounded, analytic or entire, complex solutions of the Benjamin-Bona-Mahony equation [2]

$$u_t + u_x + uu_x - u_{xxt} = 0, \quad u = u(x,t) \tag{1.1}$$

as well as the associated linear equation

$$u_t + u_x - u_{xxt} = 0, \quad u = u(x,t). \tag{1.2}$$

This technique was used for the first time in [16], for finding a necessary and sufficient condition for the existence of polynomial solutions of a class of linear partial differential equations (PDEs). Its main idea, is the transformation of the PDE into an equivalent operator equation in an abstract Hilbert or Banach space. Moreover, this technique is an extension of another functional-analytic technique for the study of analytic solutions of initial value problems of ordinary differential equations (ODEs), introduced by Ifantis [12] and systemized in [13, 14].

In the present study, the analytic solutions of the general class of nonlinear PDEs

$$u_{xt} + au_x + bu_t + cu = g(x,t) + G(u(x,t)), \quad u = u(x,t) \tag{1.3}$$

where $G(u(x,t)) = \sum_{n=2}^{\infty} c_n [u(x,t)]^n$ will be studied, extending in this way the method of [17] to other kind of nonlinear terms. It should be noted that the

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