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ANALYTICAL AND NUMERICAL STUDY OF THE FORCED BURGERS' EQUATION

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Abstract. *The problem of the forced Burgers' equation is studied using the Homotopy Analysis Method (HAM). The obtained analytical solutions are in very good agreement with corresponding numerical ones. The results are interesting, indicating a significant influence of the forcing term.*

1 INTRODUCTION

It is essential to have a deep understanding of the mechanism of fluid flow through a pipe, as it is extensively used in advanced engineering systems, in industry and in everyday life. In the human body, blood constantly flows through the arteries and veins. All modern biomedical instruments, such as artificial hearts and dialysis systems, operate on the basis of fluid flow through tubes. In refrigeration and air conditioning applications, refrigerants flow through pipes. Thus, it is important to develop and study model equations to examine certain characteristics of the processes involved in these phenomena.

One of the most famous equations used in such kind of problems, is the Burgers' equation[1]

$$u_t + \alpha u u_x - \nu u_{xx} = 0 \tag{1}$$

and the corresponding forced Burgers' equation[2]

$$u_t + \alpha u u_x - \nu u_{xx} = f(x), \tag{2}$$

where $u = u(x,t)$, $x \in (0,L)$, $t > 0$ with initial condition

$$u(x,0) = g(x). \tag{3}$$

The terms u, x, t and ν represent fluid velocity, spatial variable, time variable and kinematic viscosity of the fluid, respectively. The function $g(x)$ is a known function that satisfy suitable conditions depending on the problem to be solved and the forcing term $f(x)$ represents an external force.

Despite of the nonlinear term uu_x appearing in both equations (1) and (2), some exact solutions of them have been found[2], [3], [4], [5]. However, equations (1) and (2) are mostly studied numerically[3]. It is the aim of this presentation to show that the Homotopy Analysis Method (HAM) can be efficiently used for the study of both equations (1) and (2) and for various initial conditions and forcing terms.

2 THE HOMOTOPY ANALYSIS METHOD

The Homotopy Analysis Method (HAM) is a powerful analytical technique, which, since its appearance, has been extensively used in a variety of physical problems, including wave propagation problems. It has been