

Convexity results for the largest zero and functions involving the largest zero of q -associated polynomials

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We prove the convexity of the largest zero of the q -Lommel, the associated Al-Salam Carlitz II and the q -associated Laguerre polynomials as well as the convexity of products of certain functions with the largest zero of the q -associated Laguerre polynomials and associated Al-Salam–Carlitz II polynomials. Moreover, as a consequence of our results concerning the q -associated Laguerre polynomials, we find a recent result regarding the convexity of the function $(1/(\alpha + 1))x_{n,1}(\alpha)$, where $x_{n,1}(\alpha)$ is the largest zero of the classical Laguerre $L_n^\alpha(x)$ polynomials. The method we use is a functional analytic one based on the three-term recurrence relations that the q -associated polynomials satisfy. By use of this method, the proofs of our results are straightforward.

Keywords: Convexity; q -Associated Laguerre polynomials; Associated Al-Salam–Carlitz II polynomials; q -Lommel polynomials

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1. Introduction and main results

Recently, there has been an increasing interest in the q -orthogonal polynomials due to their applications in various areas of mathematics and physics. The q -orthogonal polynomials are the q -analogues of the classical orthogonal polynomials. Both classes of polynomials satisfy a three-term recurrence relation. However, although the classical orthogonal polynomials satisfy a second-order Sturm–Liouville type differential equation, the q -orthogonal polynomials satisfy the q -difference equation

$$\sigma(x)D_q D_{1/q}y(x) + \tau(x)D_q y(x) + \lambda_{q,n}y(x) = 0,$$

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