Integral Transforms and Special Functions, 2013
Vol. 24, No. 7, 540–547, http://dx.doi.org/10.1080/10652469.2012.720253

On the zeros of $J_\nu'''(x)$

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(*Received 18 April 2012; final version received 8 August 2012*)

The zeros of $J_\nu'''(x)$ are studied by using classical analysis and the properties of $J_\nu(x)$. It is proved that $J_\nu'''(x)$ has infinite positive zeros and between two consecutive positive zeros of $J_\nu(x)$, there exist at least one zero of $J_\nu'''(x)$ for $\nu > 1$. Moreover, several theorems are given regarding their location depending on the values of ν. Also, alternative proofs are given regarding the monotonicity of the positive zeros of $J_\nu'''(x)$ for $\nu > (1+\sqrt{5})/2$ and $\nu > 1$.

Keywords: Bessel functions; zeros; location; monotonicity

AMS Subject Classification: 33C10

1. Introduction

Bessel functions and their zeros appear naturally in various boundary value problems of partial differential equations. For several of these problems, the reader could consult several classic books such as [1,12,23,24], or refer to several research papers, some of the most recent of which are [13,21]. For an interesting electrostatic interpretation of the zeros of Bessel functions, see [22].

Thus, the study of the zeros $j_{\nu,m}$, $m = 1, 2, 3, \ldots$, of the Bessel function of first kind and order ν, $J_\nu(x)$, has been a popular subject for many decades. Indicatively, the survey papers [2,10] are mentioned. Of course, there are probably hundreds of other papers on the subject, but it is not possible to mention all of them and it would only be unfair to mention just a few.

Due to similar mathematical and physical reasons, the study of the zeros of the derivatives of $J_\nu(x)$ is also an interesting subject. The behaviour of the zeros $j'_{\nu,m}$, $m = 1, 2, 3, \ldots$, of $J'_\nu(x)$ has been studied extensively in a large number of papers. See [3,4,14,17] and the references therein, where results analogous or relevant to the results of this article are provided.

Also, some results exist for the behaviour of the zeros $j''_{\nu,m}$, $m = 1, 2, 3, \ldots$, of $J''_\nu(x)$ [5,8,16,18]. However, to the best of the authors' knowledge, there exist very few results in the literature regarding $J_\nu'''(x)$ and their zeros $j'''_{\nu,m}$, $m = 1, 2, 3, \ldots$. More precisely, the monotonicity of $j'''_{\nu,m}$ has been studied in [15,19]. As mentioned in [19], these results 'are reminiscent of the corresponding phenomena for the zeros of $J_\nu(x)$, $J'_\nu(x)$ and $J''_\nu(x)$'.

In this article, the zeros $j'''_{\nu,m}$, $m = 1, 2, 3, \ldots$ of $J_\nu'''(x)$ will be studied for $\nu \in \mathbb{R}$. This study is based on the observation that, the non-zero real zeros of $J_\nu'''(x)$, as it will be shown in Section 2,

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