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On the common zeros of Bessel functions

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Abstract

Using a functional analytic method we give some results concerning common zeros of the ordinary Bessel functions $J_\nu(z)$ of first kind, with respect to ν and fixed z. A lower bound for the common zero z of the Bessel functions $J_\nu(z)$, $J_\mu(z)$, where ν, μ are known, is also given.

MSC: 33C10

Keywords: Common zeros; Bessel functions

1. Introduction

Investigating the common zeros of Bessel functions of first kind, $J_\nu(z)$, is a quite old and important problem of Bessel functions for which various theoretical and numerical results exist. One such result is that the Bessel functions $J_\nu(z)$ and $J_\mu(z)$, where μ rational and $\nu-\mu$ is a positive integer, cannot have common zeros (Bourget's hypothesis) [14, p. 484]. A simple alternative proof of a specific case of this result was given in [9, p. 464]. However, this result does not exclude the possibility that two functions $J_\nu(z)$ and $J_\mu(z)$, where ν,μ do not differ by a positive integer, may have common zeros.

The problem of locating the zeros can be "attacked" in two ways. One way is to consider a fixed z_0 and investigate the function $J_\nu(z_0)$ as a function of ν. The first who did this was Dougall in 1900 [4] who proved that, if z_0 is purely imaginary, then the real part of ν cannot be nonnegative. Later, in 1936, Coulomb [3] improved this result by proving that for z_0 purely imaginary and ν not real, the real part of ν cannot be greater than $-\frac{3}{2}$. In the same paper he proved that there exists a

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