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Existence and uniqueness of solutions in $H_1(\Delta)$ of a general class of non-linear functional equations

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Abstract

A functional analytic method is used to prove the existence and the uniqueness of a solution in the Banach space $H_1(\Delta)$ of a general class of non-linear functional equations. This general class includes some specific functional equations studied recently. Our results simplify and improve the existing results for these specific equations. Moreover, for one of them, we give an answer to an open problem.

Keywords: Functional equations; Existence; Uniqueness; Analytic; Bounded solutions

1. Introduction

In this paper, we give sufficient conditions such that a class of non-linear functional equations has a unique analytic solution in the Banach space $H_1(\Delta)$ of analytic functions, defined as follows:

$$H_1(\Delta) = \left\{ f(z) = \sum_{n=1}^{\infty} f_n z^{n-1} \text{ analytic in } \Delta \text{ and } \sum_{n=1}^{\infty} |f_n| < +\infty \right\}, \tag{1.1}$$

where $\Delta = \{z \in \mathbb{C}/|z| < 1\}$, with norm $\|f(z)\|_{H_1(\Delta)} = \sum_{n=1}^{\infty} |f_n|$.

This class of functional equations includes as particular cases the following equations:

$$f(\lambda^2 z) = 2f(\lambda z) - f(z) - \frac{1}{2} g\big(f(\lambda z)\big) + g\big(f(z)\big), \quad z \in \mathbb{C}, \tag{1.2}$$

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