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A "discretization" technique for the solution of ODEs ☆

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Abstract

A functional-analytic technique was developed in the past for the establishment of unique solutions of ODEs in $H_2(\mathbb{D})$ and $H_1(\mathbb{D})$ and of difference equations in ℓ_2 and ℓ_1. This technique is based on two isomorphisms between the involved spaces. In this paper, the two isomorphisms are combined in order to find discrete equivalent counterparts of ODEs, so as to obtain eventually the solution of the ODEs under consideration. As an application, the Duffing equation and the Lorenz system are studied. The results are compared with numerical ones obtained using the 4th order Runge–Kutta method. The advantages of the present method are that, it is accurate, the only errors involved are the round-off errors, it does not depend on the grid used and the obtained solution is proved to be unique.

Keywords: Numerical solution; Functional-analytic method; Duffing; Lorenz

1. Introduction

In [1,2], a functional-analytic technique appeared for the study of linear ordinary differential equations (ODEs) in the Hardy–Lebesgue Hilbert space of analytic functions defined in $\mathbb{D} =$

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