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Self-adjointness of unbounded tridiagonal operators and spectra of their finite truncations

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ABSTRACT

This paper addresses two different but related questions regarding an unbounded symmetric tridiagonal operator: its self-adjointness and the approximation of its spectrum by the eigenvalues of its finite truncations. The sufficient conditions given in both cases improve and generalize previously known results. It turns out that, not only self-adjointness helps to study limit points of eigenvalues of truncated operators, but the analysis of such limit points is a key help to prove self-adjointness. Several examples show the advantages of these new results compared with previous ones. Besides, an application to the theory of continued fractions is pointed out.

1. Introduction

Symmetric tridiagonal matrices provide the canonical matrix representations of self-adjoint operators in Hilbert spaces [25] and, as a consequence, they naturally emerge in phenomena governed by self-adjoint operators. On the other hand, self-adjoint operators are ubiquitous in practical applications because of the usual requirement of a real spectrum in physical problems. Due to these reasons, symmetric tridiagonal operators appear in many areas of mathematics and physics.

A symmetric tridiagonal operator T in an infinite dimensional Hilbert space $(H, (\cdot,\cdot))$ with an orthonormal basis $\{e_n\}_{n=1}^{\infty}$ is given without loss by

$$Te_n = a_n e_{n+1} + b_n e_n + a_{n-1} e_{n-1}, \quad a_n > 0,\ b_n \in \mathbb{R},\ n = 1, 2, \ldots, \tag{1.1}$$

where $e_0 = 0$. The matrix representation of T in the basis $\{e_n\}_{n=1}^{\infty}$ is

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