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Exact solutions of Burgers equation with moving boundary

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ABSTRACT

In this paper, new symmetry reductions and similarity solutions for Burgers equation with moving boundary are obtained by means of Lie's method of infinitesimal transformation groups, for a linearly moving boundary as well as a parabolically moving boundary. By using discrete symmetries, new analytical solutions for the problem under consideration are presented, for two cases of the moving boundary: one moving with constant velocity and another one rapidly oscillating.

I. INTRODUCTION

The Burgers equation

$$u_t = (\nu u_x - \sigma u^2)_x, \quad u \equiv u(x,t) \tag{1}$$

is well-known for its theoretical and practical interest connected with fluid mechanics problems (one may refer to Sachdev[1] and Whitham[2]). In Eq. (1), ν, σ are non zero parameters, whereas $u(x,t)$ stands for the velocity of the fluid, with x and t being the spatial and time coordinate, respectively. Since the values of ν and σ can be modified via rescaling, it suffices to consider only the case $\nu = 1$.

It is well known, that the initial value problem associated with Eq. (1) can be easily solved by reducing (1) to the linear heat equation, using the Hopf-Cole transformation (see for example Whitham[2]). However, solving boundary value problems for Eq. (1) is more complicated and they have motivated several studies in the past, both theoretical and numerical. From a theoretical point of view, semi line solutions and solutions on a finite interval of Eq. (1) were obtained for the initial/boundary problem with a time dependent boundary condition by Calogero and de Lillo[3,4] (for the case of a time-independent boundary condition on the semiline, one may refer to Joseph[5]). The equivalence of such solutions to solutions of the forced Burgers equation in the case of an additive forcing term of distribution type was established by Ablowitz and de Lillo.[6,7] Moreover, Calogero and de Lillo[8] solved, on the semiline, the initial/boundary value problem for Eq. (1) with a general boundary condition, by reducing it to a one variable equation. In the case of flux-type boundary conditions, Joseph and Sachdev,[9] Biondini and de Lillo[10] and Besong[11] obtained explicit solutions for Eq. (1).

Numerical solutions for boundary value problems associated with Eq. (1) are also reported in the literature. For example, Zaki[12] obtained numerical solutions of the Korteweg–De Vries–Burgers equation implementing a cubic B-spline finite element method based on Bubnov–Galerkin's method. Dağ *et al.*[13] found numerical solutions of the Burgers equation using the first order splitting method combined with quadratic and B-spline Galerkin methods. Xie *et al.*[14] presented a numerical method for solving the one-dimensional Burgers' equation