Journal of Difference Equations and Applications,
Vol. 11, No. 1, January 2005, 49–62

Taylor & Francis
Taylor & Francis Group

Existence of complex ℓ_2 solutions of linear delay systems of difference equations

E. N. PETROPOULOU† and P. D. SIAFARIKAS*‡

†Department of Engineering Sciences, Division of Applied Mathematics and Mechanics, University of Patras, 26500, Patras, Greece
‡Department of Mathematics, University of Patras, 26500, Patras, Greece

(Received 10 December 2003; revised 11 May 2004; in final form 18 August 2004)

We give sufficient conditions for the existence of complex ℓ_2 solutions of a non-homogeneous system of linear difference equations and of two general classes of delay systems of linear difference equations. In some cases, bounds of the established solutions are also given. As a consequence of the space ℓ_2 where we work, information can be obtained about the asymptotic behavior of the established solutions and, the asymptotic stability of the zero equilibrium point of the systems under consideration. The method we use is a functional-analytic one.

Keywords: Delay; ℓ_2 solutions; Existence; Difference equations

Mathematics Subject Classification: 39A11; 39B72

1. Introduction and main results

The aim of this paper is to study the existence of complex ℓ_2 solutions of the following:

(a) non-homogeneous linear system of difference equations

$$\left.\begin{array}{l} f(n+1) = A(n)f(n) + g(n), \quad n = 1, 2, \ldots, \\ f(1) \text{ known initial condition} \end{array}\right\}, \tag{1.1}$$

where $f(n) = (f_1(n), \ldots, f_k(n)), g(n) = (g_1(n), \ldots, g_k(n))$ are elements of $\ell_2^k = \underbrace{\ell_2 \times \ldots \times \ell_2}_{k-\text{times}}$ and $A(n) = (a_{ij}(n))$ a $k \times k$ matrix of complex sequences, k a finite positive integer.

(b) Delay, homogeneous, linear system of difference equations

$$\left.\begin{array}{l} f(n+1) = \Lambda(n)f(n) + \sum_{r=1}^{R} A_r(n)f(n-r), \quad n = 1+R, 2+R, \ldots, \\ f(1), f(2), \ldots, f(1+R) \text{ known initial conditions} \end{array}\right\}, \tag{1.2}$$

where $f(n) = (f_1(n), \ldots, f_k(n))$, $\Lambda(n) = (\lambda_{ij}(n))$ and $A_r(n) = \left(a_{ij}^{(r)}(n)\right)$, $k \times k$ matrices of complex sequences, k a finite positive integer and $R \in \mathbb{N}$, $R \geq 1$.

(c) Delay, non-homogeneous, linear system of difference equations

$$(n - d_i)f_i(n - d_i + 1) = a_{i1}(n)f_1(n) + \ldots + a_{ik}(n)f_k(n) + g_i(n), \tag{1.3}$$

*Corresponding author. Email: panos@math.upatras.gr

Journal of Difference Equations and Applications
ISSN 1023-6198 print/ISSN 1563-5120 online © 2005 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/10236190412331313386