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On the eigenvalue problem of a class of linear partial difference equations

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In this paper, the eigenvalue problem of a class of linear partial difference equations is studied. The results concern the existence of eigenvalues, their character (real, positive), as well as the behavior of its eigenfunctions (positivity, oscillation). Moreover a theorem is given concerning the existence of a unique solution of an associated non-homogeneous partial difference equation. The results generalize previously known results for ordinary linear difference equations. The method used is a functional-analytic one, which transforms the eigenvalue problem for the difference equation into the equivalent problem of the eigenvalues of an operator defined on an abstract separable Hilbert space.

Keywords: linear partial difference equations; eigenvalues; eigenfunctions

AMS Subject Classification: 39A11; 39A70; 47N99

1. Introduction

Lately, there has been an increasing interest in the study of partial difference equations due not only to mathematical reasons, but also due to the increasing development of computers and numerical techniques, as well as their applications in various sciences, such as biology and physics. A large number of papers have appeared where several methods were developed for the study of partial difference equations, as well as monographs devoted exclusively to partial difference equations, such as [4]. Methods for the study of partial difference equations have also been included in several books regarding difference equations, such as [2,10,11].

Many of the research results on partial difference equations concern the asymptotic behavior of their solutions, as well as the positivity, bounded or oscillatory character of them. Also, the study of the eigenvalue problem of linear partial difference equations is quite important, since such kind of problems appear often in the numerical solution of partial differential equations using finite difference methods. For details on the numerical solution of partial differential equations, one may consult [3] or [12].

From a purely theoretical point of view a result related to the present paper is the following theorem presented in Ref. [4] (Theorem 115, p. 215, see also pp. 17, 18 and 23 for the notation):

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