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ON THE LOGISTIC EQUATION IN THE COMPLEX PLANE

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□ *The famous logistic differential equation is studied in the complex plane. The method used is based on a functional analytic technique which provides a unique solution of the ordinary differential equation (ODE) under consideration in $H_2(\mathbb{D})$ or $H_1(\mathbb{D})$ and gives rise to an equivalent difference equation for which a unique solution is established in ℓ_2 or ℓ_1. For the derivation of the solution of the logistic differential equation this discrete equivalent equation is used. The obtained solution is analytic in $\{z \in \mathbb{C} : |z| < T\}$, $T > 0$. Numerical experiments were also performed using the classical 4th order Runge–Kutta method. The obtained results were compared for real solutions as well as for solutions of the form $y(t) = u(t) + iv(t)$, $t \in \mathbb{R}$. For $t \in \mathbb{C}$ the solution derived by the present method, seems to have singularities, that is, points where it ceases to be analytic, in certain sectors of the complex plane. These sectors, depending on the values of the involved parameters, can move at different directions, join forming common sectors, or pass through each other and continue moving independently. Moreover, the real and imaginary part of the solution seem to exhibit oscillatory behavior near these sectors.*

Keywords Complex solution; Functional-analytic method; Logistic; Singularities.

Mathematics Subject Classification 34A12; 34A25; 34A34; 34M10; 34M35; 65L05.

1. INTRODUCTION

The logistic differential equation

$$y'(t) = \beta y(t) - \gamma[y(t)]^2, \quad y(0) = a, \tag{1.1}$$

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