E. N. Petropoulou, Study of Difference Equations in Hilbert and Banach Spaces and their Applications, Department of Mathematics, University of Patras, Patras, 2002 (200 pages) (in greek).

In this thesis, we study the solutions of:
a) **non-linear ordinary** difference equations in the Banach space

$$\ell^1_{\mathbb{N}} = \{f(n) : \mathbb{N} \to \mathbb{C} / \sum_{n=1}^{\infty} |f(n)| < +\infty\},$$

b) **linear** and **non-linear** partial difference equations of **two** variables in the Hilbert space

$$\ell^2_{\mathbb{N}\times\mathbb{N}} = \{u(i,j) : \mathbb{N}\times\mathbb{N} \to \mathbb{C} / \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} |u(i,j)|^2 < +\infty\}$$

and the Banach space

$$\ell^1_{\mathbb{N}\times\mathbb{N}} = \{u(i,j) : \mathbb{N}\times\mathbb{N} \to \mathbb{C} / \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} |u(i,j)| < +\infty\},$$

respectively,
c) **linear** and **non-linear** partial difference equations of **three** variables in the Hilbert space

$$\ell^2_{\mathbb{N}^3} = \{u(i,j,k) : \mathbb{N}^3 \to \mathbb{C} / \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} |u(i,j,k)|^2 < +\infty\}$$

and the Banach space

$$\ell^1_{\mathbb{N}^3} = \{u(i,j,k) : \mathbb{N}^3 \to \mathbb{C} / \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} |u(i,j,k)| < +\infty\},$$

respectively and
d) **non-linear** partial difference equations of **four** variables in the Hilbert space

$$\ell^1_{\mathbb{N}^4} = \{u(i,j,k,l) : \mathbb{N}^4 \to \mathbb{C} / \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} |u(i,j,k,l)| < +\infty\}.$$

For the study of ordinary difference equations we use a functional-analytic method, introduced by E. K. Ifantis in [E. K. Ifantis, On the convergence of power-series whose coefficients satisfy a Poincaré type linear and nonlinear difference equation, *Complex Variables*, **9** (1987), 63-80.], but we extend his method so as to include non-homogeneous terms and more general non-linear terms. For the study of partial difference equations of p variables ($p \in \mathbb{N}$, $p \geq 2$), we introduce a new functional-analytic method. In both cases, the functional-analytic method which we use, enables us to transform the difference equation under consideration, **equivalently**, into an operator equation in an abstract separable Hilbert space H or an abstract Banach space H_1.

More precisely, we give necessary conditions, regarding the non-homogeneous term, the parameters and the non-linear terms, so as the following general class of non-linear ordinary difference equation:

$$f(n+m)+\sum_{p=1}^{m}(\alpha_p+\beta_p(n))f(n+m-p)=g(n)+\sum_{s=2}^{\infty}c_s(n)[f(n+q)]^s+$$

$$+\sum_{i=1}^{N}\sum_{k=1}^{\infty}d_{ik}(n)[f(n+q_{i1})f(n+q_{i2})]^k+$$

$$+\sum_{t=1}^{\Lambda}\sum_{k=1}^{\infty}b_{tk}(n)[f(n+q_{t3})f(n+q_{t4})f(n+q_{t5})]^k+$$

$$+\sum_{j=1}^{M}\sum_{k=1}^{\infty}l_{jk}(n)[A_jf(n+q_{j6})+B_jf(n+q_{j7})]^kf(n+q_{j8}), \qquad (1)$$

together with complex initial conditions, to have a unique solution in the Banach space $\ell^1_{\mathbb{N}}$. Moreover, we give a bound of the solution and a region, depending on the initial conditions, the non-homogeneous term and the parameters of the equation, where this solution holds. This region can be considered as a region of attraction for the zero equilibrium point of (1), because the predicted solution belongs to $\ell^1_{\mathbb{N}}$ and thus it will tend to zero, as n tends to infinity. Thus we obtain information regarding the asymptotic behavior of the solutions of (1). Also, if ϱ is a non-zero equilibrium point of (1), it can be proved by using the transformation $f(n)=F(n)+\varrho$, that under appropriate conditions, equation (1) has a unique solution in $\ell^1_{\mathbb{N}}+\{\varrho\}$. Some known non-linear ordinary difference equations, some of which appear in problems of population dynamics, are studied as particular cases of (1). However, one of them cannot be studied as a particular case of (1) and thus we adjust our method appropriately, so as to study also this difference equation.

Also, we give necessary conditions, regarding the non-homogeneous term and the parameters, so as the following general class of linear partial difference equation of two variables:

$$u(i,j+1) = p(i,j)+\sum_{n=1}^{N} \alpha_n(i,j)u(i-\sigma_{n1},j-\tau_{n1})+\sum_{m=1}^{M} b_m(i,j)u(i+\sigma_{m2},j+\tau_{m2}) \tag{2}$$

together with complex initial conditions, to have a unique solution in the Hilbert space $\ell^2_{\mathbb{N}\times\mathbb{N}}$. Moreover, we give a bound of the solution. Since the predicted solution belongs to $\ell^2_{\mathbb{N}\times\mathbb{N}}$, it will tend to zero, as i,j tend to infinity. Thus we obtain information regarding the asymptotic behavior of the solutions of (2). Some known linear partial difference equations of two variables, most of which appear in problems of physics, population dynamics and numerical schemes, are studied as particular cases of (2). However, one of them cannot be studied as a particular case of (2) and thus we adjust our method appropriately, so as to study also this difference equation.

We also give necessary conditions, regarding the non-homogeneous term, the parameters and the non-linear terms, so as the following general class of non-linear partial difference equation of two variables:

$$\sum_{n=1}^{N} \alpha_n(i,j)u(i-\sigma_{n1},j-\tau_{n1}) + \sum_{m=1}^{M} b_m(i,j)u(i+\sigma_{m2},j+\tau_{m2})+$$
$$+\sum_{k=1}^{K} c_k(i,j)u(i-\sigma_{k3},j+\tau_{k3}) + \sum_{l=1}^{\Lambda} d_l(i,j)u(i+\sigma_{l4},j-\tau_{l4}) = p(i,j)+$$
$$+\sum_{s=2}^{\infty} f_s(i,j)[u(i+\sigma,j+\tau)]^s+\sum_{t=1}^{T} q_t(i,j)u(i+\sigma_{t5},j+\tau_{t5})u(i+\sigma_{t6},j+\tau_{t6}), \tag{3}$$

together with complex initial conditions, to have a unique solution in the Banach space $\ell^1_{\mathbb{N}\times\mathbb{N}}$. Moreover, we give a bound of the solution and a region, depending on the initial conditions, the non-homogeneous term and the parameters of the equation, where this solution holds. Since the predicted solution belongs to $\ell^1_{\mathbb{N}\times\mathbb{N}}$, it will tend to zero, as i,j tend to infinity. Thus we obtain information regarding the asymptotic behavior of the solutions of (3).

Some known non-linear partial difference equations of two variables, most of which appear in problems of population dynamics and numerical schemes, are studied as particular cases of (3). However, some of them cannot be studied as a particular case of (3) and thus we adjust our method appropriately, so as to study also these difference equations.

Finally, we study a linear partial difference equation of three variables that appear in a problem of physics, a non-linear partial difference equation of three variables that appear in a problem of numerical schemes and, a non-linear partial difference equation of four variables that appear in a problem of game theory. For each one of them, we prove that, under appropriate necessary conditions, they have a unique solution in the spaces $\ell^2_{\mathbb{N}^3}$, $\ell^1_{\mathbb{N}^3}$ and $\ell^1_{\mathbb{N}^4}$, respectively. Also for these non-linear partial difference equations, we give a region, depending on the initial conditions (which are considered in general complex), the non-homogeneous term and the parameters of the equations, where the predicted solution holds.