

Qualitative, Approximate and Numerical Approaches for the Solution of Nonlinear Differential Equations



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1 Introduction

During a standard undergraduate course in differential equations (DEs), ordinary (ODEs) or partial (PDEs), we are taught several methods in order to find their exact solutions, i.e. their solutions in terms of elementary or special functions, or even in the form of power series in their independent variable(s), provided that the corresponding coefficients can be uniquely determined. All these methods refer mostly to linear equations and a “small” amount of specific classes of nonlinear equations. Unfortunately, most differential equations describing real life problems are nonlinear and the vast majority of these cannot be solved explicitly. In these cases, we are obliged to use other, more advanced techniques widely used in research, some of which are also taught in some undergraduate and many postgraduate courses. In several cases not even these methods can give satisfactory results and the need for new methods is required.

The aim of this chapter is to describe some of these more advanced techniques which can be characterized as (a) qualitative, (b) approximate or (c) numerical. Qualitative methods are employed in order to obtain information about the qualitative characteristics of the solution of a DE. As a consequence, they provide answers to questions such as:

- Do they exist solutions in specific spaces of interest? If yes, are they unique?
- Is the solution bounded? Can we find explicit bounds for it?

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