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Solving the Nonlinear Boundary Layer Flow Equations with Pressure Gradient and Radiation

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Abstract: The physical problem under consideration is the boundary layer problem of an incompressible, laminar flow, taking place over a flat plate in the presence of a pressure gradient and radiation. For the mathematical formulation of the problem, the partial differential equations of continuity, energy, and momentum are taken into consideration with the boundary layer simplifications. Using the dimensionless Falkner–Skan transformation, a nonlinear, nonhomogeneous, coupled system of partial differential equations (PDEs) is obtained, which is solved via the homotopy analysis method. The obtained analytical solution describes radiation and pressure gradient effects on the boundary layer flow. These analytical results reveal that the adverse or favorable pressure gradient influences the dimensionless velocity and the dimensionless temperature of the boundary layer. An adverse pressure gradient causes significant changes on the dimensionless wall shear parameter and the dimensionless wall heat-transfer parameter. Thermal radiation influences the thermal boundary layer. The analytical results are in very good agreement with the corresponding numerical ones obtained using a modification of the Keller's-box method.

Keywords: boundary layer; approximate solution; homotopy analysis method; pressure gradient; thermal radiation.

MSC: 76D10; 76M99; 35C10; 35Q35

1. Introduction

The mathematical description of the laminar, incompressible boundary layer flow remains a challenging problem with several aspects still unexplored. The first to describe the steady two-dimensional boundary layer flow over a semi-infinite flat plate was Blasius [1]. The adverse pressure gradient on the laminar boundary layer flow was numerically examined by Howarth [2]. Other studies on the effects of adverse pressure gradients, heat transfer, or thermal radiation on the fluid flow include, although not limited to, [3–8]. The effect of radiation in the presence of a magnetic field or with temperature dependent viscosity was studied in [9,10].

At high temperatures, thermal radiation has a significant effect on the flow field, which has important applications in many areas of engineering. This effect has been reported in several works